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XV. *Remarks on the* Hadley's *Quadrant, tending principally to remove the Difficulties which have hitherto attended the Use of the Back-observation, and to obviate the Errors that might arise from a Want of Parallelism in the two Surfaces of the Index-Glass. By* Nevil Maskelyne, *F. R. S. Astronomer Royal.*

Read May 28, 1772.

THE back observation with Hadley's quadrant being founded on the same principles, and in theory, equally perfect with the fore-observation, and being at the same time necessary to extend the use of the instrument up to 180 degrees (it being impracticable to measure angles with any convenience beyond 120 degrees with the fore-observation) it may seem surprizing that it hath not been brought equally into general use, more especially since the method of finding the longitude by observations of the Moon, has been practised at sea for some years past; since this method would receive considerable advantage from the use of the back-observation in taking distances of the Sun and Moon between the first and last quarter, could such

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obſervations be as much depended upon as the fore obſervations. The cauſes of this ſeem to have been principally theſe two, the difficulty of adjuſting the back horizon glaſs, and the want of a method of directing the ſight parallel to the plane of the quadrant. The back horizon glaſs, like the fore-one, requires two adjuſtments; the firſt, or common one, diſpoſes it at right angles to the index glaſs, when the index ſtands at (o) upon the arch; which is uſually performed by ſetting (o) of the index of the arch of the quadrant by double the dip of the horizon of the ſea, and then holding the quadrant vertical with the arch downwards, and turning the back-horizon glaſs about, by means of its lever or perpetual ſcrew, till the reflected back horizon appears to coincide with the fore-horizon ſeen directly. But this operation is ſo difficult in practice with the back-horizon glaſs wholly ſilvered, except a ſmall tranſparent ſlit in the middle, as it has been uſually made, that few (if any) perſons have ever received proper ſatisfaction from it. If the back-horizon-glaſs was ſilvered in every reſpect like the fore-horizon-glaſs (which it ought to be) the upper part being left unſilvered, and a teleſcope was applied to it, perhaps this adjuſtment might be rendered ſomewhat eaſier and more exact; but it could not even thus be made ſo exact as the adjuſtment of the fore-horizon-glaſs may, by making uſe of the Sun's limbs.

The ſecond adjuſtment of the back-horizon-glaſs, in the common conſtruction of the quadrant, is ſtill more troubleſome, ſince it cannot be executed without ſetting the index 90 degrees off the arch, in order to place the index-glaſs par-

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allel to the back-horizon-glaſs; when this adjuſtment may be performed in the ſame manner as the correſponding adjuſtment of the fore-horizon-glaſs. But the bending of the index, that follows the ſetting it off the arch, is a very diſagreeable circumſtance, having a tendency, eſpecially on board of ſhip, to expoſe both the index and centre work to damage; and may even, without extraordinary precautions taken by the inſtrument maker in placing the plane of the index-glaſs exactly according to the length of the index, diſturb its perpendicularity to the plane of the quadrant: on theſe accounts it would be much better if this adjuſtment of the back-horizon-glaſs could be performed, like thoſe of the fore-horizon-glaſs, with the index remaining upon the arch of the quadrant. Fortunately, this *deſideratum* has been lately effected by an ingenious contrivance invented by Mr. Dollond, which he has given an account of in a letter addreſſed to me *, which I have preſented to this Society, by means of an additional index applied to the back-horizon-glaſs; whereby both the adjuſtments may be made by the ſame obſervations and with nearly the ſame exactneſs as thoſe of the fore-horizon-glaſs: for a farther knowledge of which, ſee the account itſelf.

Beſides the difficulty of adjuſting the back-horizon-glaſs, the want of a method of directing the line of ſight parallel to the plane of the quadrant has proved alſo a conſiderable obſtacle to the uſe of the back-obſervation: this will eaſily appear from the following propoſition, that the error of the angle meaſured ariſing from any ſmall de-

* See the XIVth paper, which immediately precedes this.

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viation of the visual ray from a parallelism to the plane of the quadrant, is to twice an arch equal to the verse-sine of the deviation; as the tangent of half the angle measured by the quadrant is to radius, very nearly. Thus a deviation of 1° in the line of sight, will produce an error of about 1′ in measuring an angle of 90°, whether by the fore or back observation; but the same deviation will produce an error of 4′ in measuring an angle of 150°, of 6′ in taking an angle of 160°, and 12′ in taking an angle of 170°. Hence a pretty exact adjustment of the line of sight, or axis of the telescope, is requisite in measuring large angles, such as those are taken by the back observation: and therefore a director of the sight ought by no means to be omitted in the construction of the instrument (as it commonly has been since Mr. Hadley's time, though recommended by him), except a telescope be made use of, which if rightly placed answers the same purpose better, especially in observing the distance of the Moon from the Sun between the first and last quarter. The director of the sight may be placed exact enough by construction; but the telescope cannot, and Mr. Hadley, not having been aware of the importance of an exact position of it, has accordingly given no directions for the placing it. I shall therefore endeavour to supply this defect in the following remarks.

In the first place, I would by all means recommend an adjusting piece to be applied to the telescope, whereby its axis may be brought parallel to the plane of the quadrant: in the next place, the back-horizon-glass ought to be silvered in the same manner as the fore-horizon-glass: and thirdly, two thick silver wires should be placed within the eye-tube

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in the focus of the eye-glaſs parallel to one another, and to the plane of the quadrant. If they were put at ſuch a diſtance as to divide the diameter of the field of view into three equal parts, it might be as convenient as any other interval. In this manner wires were placed in the teleſcope by Mr. Hadley, as appears by his account of the inſtrument in Philoſ. Tranſ. N° 420. Theſe wires are to be adjuſted parallel to the plane of the quadrant, by turning the eye-tube round about which contains the wires, till they appear parallel to the plane of the quadrant. The axis of the teleſcope, by which is meant the line joining the centre of the object-glaſs and the middle point between the two wires, is to be adjuſted parallel to the plane of the quadrant by either of the two following methods.

1ſt method. When the diſtance of the Moon from the Sun is greater than 90 degrees, by giving a ſweep with the quadrant and moving the index, bring the neareſt limbs to touch one another at the wire neareſt the plane of the quadrant. Then, the index remaining unmoved, make the like obſervation at the wire fartheſt from the plane of the quadrant; and note whether the neareſt limbs are in contact as they were at the other wire: if they are, the axis of the teleſcope is parallel to the plane of the quadrant: but if they are not, it is inclined to the ſame, and muſt be corrected as follows. If the neareſt limbs of the Sun and Moon ſeem to lap over one another at the wire fartheſt from the plane of the quadrant, the object end of the teleſcope is inclined from the plane of the quadrant, and muſt be altered by the adjuſtment made for that purpoſe: but, if the neareſt limbs

limbs of the Sun and Moon do not come to touch one another at the wire fartheſt from the plane of the quadrant, the object end of the teleſcope is inclined towards the plane of the quadrant, and muſt be altered by the adjuſtment accordingly. Let theſe operations be repeated until the obſervation is the ſame at both the parallel wires, and the axis of the teleſcope will be adjuſted parallel to the plane of the quadrant. In like manner, the axis of the teleſcope may be alſo adjuſted parallel to the plane of the quadrant for the fore-obſervation.

Second method. Set the index to (o) and hold the plane of the quadrant parallel to the horizon of the ſea, with the divided arch upwards, the two wires being parallel to, and including both the direct fore-horizon, and the reflected back-horizon, between them. Raiſe or lower the plane of the quadrant until the direct and reflected horizons coincide together: if the coincidence happens in the middle between the two wires, or rather, to be more exact, above the middle by ſuch a part of the field of view as anſwers to the number of minutes in the depreſſion of the horizon (which may be eaſily eſtimated if the angular interval of the wires be firſt found by experiment, in manner hereafter mentioned) the axis of the teleſcope is parallel to the plane of the quadrant; but if it does not, the line of ſight is inclined to the plane of the quadrant, and muſt be corrected as follows. If the direct and reflected horizons, when they coincide, appear higher above the middle between the wires, than what the quantity of the depreſſion of the horizon amounts to, the object end of the teleſcope is inclined from the plane of the quadrant, and muſt be altered by the adjuſtment made for that purpoſe; but

but if the two horizons appear to coincide in a lower part of the field of the telescope, the object end of the telescope is inclined towards the plane of the quadrant, and must be altered by the adjustment accordingly. Repeat these operations till the two horizons appear to coincide above the middle between the two wires, by the quantity of the depression of the horizon, and the axis of the telescope will be adjusted parallel to the plane of the quadrant. In order to find the angular interval between the wires, hold the quadrant perpendicular to the horizon, as in observing altitudes; and turn about the eye-tube with the wires until they are parallel to, and include, the direct fore-horizon and reflected back-horizon between them. Move the index from (o) along the divided arch, at the same time raising or lowering the telescope by the motion of the quadrant until the direct horizon appears to coincide with the upper wire, and the reflected back-horizon with the lower wire; the number of degrees and minutes shewn upon the arch, increased by double the depression of the horizon, will be the angular interval of the wires; its proportion to the depression of the horizon will be therefore known; and hence the space in the field of the telescope answering to the depression of the horizon, may be easily estimated near enough for adjusting the axis of the telescope in the manner before-mentioned. The first of the two methods here given for adjusting the position of the telescope will probably be found most convenient; and the greater the distance of the Sun and Moon is, the more nearly may the adjustment be made, because the same deviation of the axis of the telescope will cause a greater error.

The teleſcope ſhould be fixed by the inſtrument-maker ſo as to command a full field of view when the inſtrument is placed at 90° if the inſtrument be an octant, or 120° if it be a ſextant; becauſe the index-glaſs then ſtands more oblique with reſpect to the incident and reflected rays, and conſequently the field of view of the teleſcope, as far as it depends upon the index-glaſs, will be more contracted than in any other poſition of the index: but if there is a fair field of view in this caſe, there neceſſarily muſt be ſo in every other poſition of the index.

The two parallel wires will be very uſeful on many occaſions, as well in the fore as the back-obſervation. In taking the altitude of the Sun, Moon, or ſtar, direct the ſight towards the part of the horizon underneath, or oppoſite to the object, according as you intend to obſerve by the fore or back obſervation, and hold the quadrant that the wires may conſtantly appear perpendicular to the horizon, and move the index till you ſee the object come down towards the horizon in the fore-obſervation, or up to it in the back-obſervation, and turn the inſtrument in order to bring the object between the wires; then move the index till the Sun or Moon's limb, or the ſtar touch the horizon. The nearer the object is brought to an imaginary line in the middle between the wires (it is indifferent what part of the line it is brought to) and the truer the wires are kept perpendicular to the horizon, the more exact will the obſervation be. In the fore-obſervation, the object appears in its real poſition; but in the back-obſervation, the object being brought through the zenith to the horizon, the real upper-limb will appear the loweſt;

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and the contrary. Either limb of the Sun may be used in either obſervation; but it will be moſt convenient in general to make the Sun appear againſt the ſky, and not againſt the ſea; and then the objects appearing inverted through the teleſcope, the Sun will appear loweſt, and the horizon higheſt. The obſerved altitude is to be corrected for dip, refraction, and Sun's ſemi-diameter, as uſual.

In taking the diſtance of the neareſt limbs of the Sun and Moon, whether by the fore or back-obſervation, having firſt ſet the index to the diſtance nearly, by the help of the Nautical Almanac, and brought the Moon to appear anywhere on or near the diameter of the field of view of the teleſcope, which biſects the interval between the wires, give a ſweep with the quadrant, and the Sun and Moon will paſs by one another; if in this motion the neareſt limbs, at their neareſt approach, juſt come to touch one another, without lapping over, on or near any part of the diameter of the field of the teleſcope which biſects the interval between the wires, the index is rightly ſet; but if the neareſt limbs either do not come to meet, or lap over one another, alter the index, and repeat the obſervation till the neareſt limbs come to touch one another properly. This method of obſerving will be found much more eaſy and expeditious than without the wires, ſince in that caſe it would be neceſſary to make the limbs touch very near the centre of the teleſcope, but here it is only neceſſary to make them do ſo anywhere on or near the diameter of the field of the teleſcope which biſects the interval between the two wires.

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The ſame method may be uſed in taking the Moon's diſtance from a fixed ſtar.

It may not be amiſs here to make ſome remarks on the rules that have been uſually given for obſerving the Sun's altitude, both with the fore and back-obſervation, which have all been defective, and to point out the proper directions to be followed, when a teleſcope is not uſed with two parallel wires to direct the quadrant perpendicular to the horizon, and to ſhew the principles on which theſe directions are founded.

Obſervers are commonly told, that in making the fore obſervation they ſhould move the index to bring the Sun down to the part of the horizon directly beneath him, and turn the quadrant about upon the axis of viſion; and when the Sun touches the horizon at the loweſt part of the arch deſcribed by him the quadrant will ſhew the altitude above the viſible horizon. I allow that this rule would be true, if a perſon could by ſight certainly know the part of the horizon exactly beneath the Sun; but, as this is impoſſible, the precept is incomplete. Moreover, in taking the Sun's altitude in or near the zenith, this rule entirely fails, and the beſt obſervers adviſe to hold the quadrant vertical, and turn one's ſelf about upon the heel, ſtopping when the Sun glides along the horizon without cutting it: and it is certain that this is a good rule in this caſe, and capable with care of anſwering the intended purpoſe. We have thus two rules for the ſame thing, which is a proof that neither of them is an univerſal one, or ſufficient in all caſes alone.

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In taking the back-obſervation, obſervers have been adviſed either to turn the quadrant about upon the axis of viſion, or, holding the quadrant upright, to turn themſelves about upon the heel, indifferently. The true ſtate of the caſe is this; that, in taking the Sun's altitude, whether by the fore or back-obſervation, theſe two methods muſt be combined together; that is to ſay, the obſerver muſt turn the quadrant about upon the axis of viſion, and at the ſame time turn himſelf about upon his heel, ſo as to keep the Sun always in that part of the horizon-glaſs which is at the ſame diſtance as the eye from the plane of the quadrant: for, unleſs the caution of obſerving the objects in the proper part of the horizon-glaſs be attended to, it is evident the angles meaſured cannot be true ones. In this way the reflected Sun will deſcribe an arch of a parallel circle round the true Sun, whoſe convex ſide will be downwards in the fore-obſervation, and upwards in the back-obſervation, and conſequently, when, by moving the index, the loweſt point of the arch in the fore-obſervation, or the uppermoſt point of the arch in the back-obſervation, is made to touch the horizon, the quadrant will ſtand in a vertical plane, and the altitude above the viſible horizon will be properly obſerved.

The reaſon of theſe operations may be thus explained: the image of the Sun being always kept in the axis of viſion, the index will always ſhew on the quadrant the diſtance between the Sun and any object ſeen directly which its image appears to touch; therefore, as long as the index remains unmoved, the image of the Sun will deſcribe an arch everywhere equidiſtant from the Sun in the heavens, and conſe-

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quently a parallel circle about the Sun, as a pole; ſuch a tranſlation of the Sun's image can only be produced by the quadrant being turned about upon a line drawn from the eye to the Sun, as an axis; a motion of rotation upon this line may be reſolved into two, one upon the axis of viſion, and the other upon a line on the quadrant perpendicular to the axis of viſion; and conſequently a proper combination of theſe two motions will keep the image of the Sun conſtantly in the axis of viſion, and cauſe both jointly to run over a parallel circle about the Sun in the heavens; but when the quadrant is vertical a line thereon perpendicular to the axis of viſion becomes a vertical axis; and, as a ſmall motion of the quadrant is all that is wanted, it will never differ much in practice from a vertical axis; therefore the obſerver, by properly combining and proportioning two motions, one of the quadrant upon the axis of viſion, and the other of himſelf upon his heel, keeping himſelf upright (which gives the quadrant a motion upon a vertical axis) will cauſe the image of the Sun to deſcribe a ſmall arch of a parallel circle about the Sun in the heavens, without departing conſiderably from the axis of viſion.

If it ſhould be aſked, why the obſerver ſhould be directed to perform two motions rather than the ſingle one equivalent to them on a line drawn from the eye to the Sun as an axis? I anſwer, that we are not capable, while looking towards the horizon, of judging how to turn the quadrant about upon the elevated line going to the Sun as an axis, by any other means than by combining the two motions above-mentioned, ſo as to keep the Sun's image always

ways in the proper part of the horizon-glaſs. When the Sun is near the horizon, the line going from the eye to the Sun will not be far removed from the axis of viſion; and conſequently the principal motion of the quadrant will be performed on the axis of viſion, and the part of the motion made on the vertical axis will be but ſmall. On the contrary, when the Sun is near the zenith, the line going to the Sun is not far removed from a vertical line, and conſequently the principal motion of the quadrant will be performed on a vertical axis, by the obſerver's turning himſelf about, and the part of the motion made on the axis of viſion will be but ſmall. In intermediate altitudes of the Sun, the motions of the quadrant on the axis of viſion and on a vertical axis will be more equally divided. Hence appears the reaſon of the method uſed by the beſt obſervers in taking the Sun's altitude when near the zenith by holding the quadrant vertical and turning about upon the heel, and the defects of the rules that have been commonly given for obſerving altitudes in other caſes.

As it may conduce to the ſetting this matter in a ſtill clearer light, I ſhall here deſcribe in order the ſeveral motions that will be given to the reflected image, by turning the quadrant about upon the axis of viſion, a vertical axis, or the line drawn from the eye to the Sun, ſucceſſively.

I. If the quadrant is turned about upon the axis of viſion, the ſame being directed to the point of the horizon exactly beneath or oppoſite the Sun, the image of the Sun will move from right to left, or from left to right, acroſs the

the horizon-glaſs, the ſame way as the arch of the quadrant is carried, both in the fore and back-obſervations, with a velocity which is to the angular velocity of the quadrant as the ſine of the Sun's altitude to the radius, deſcribing an arch convex downwards in both caſes; and when the motion of the Sun in this arch is parallel to the horizon, the quadrant is held truely perpendicular to the horizon, and conſequently in a proper poſition for taking the Sun's altitude. But, if the axis of viſion be directed to, and turned round a point in the horizon beſide the vertical circle paſſing through the Sun, the Sun's image, when its motion is parallel to the horizon, will be neither in the axis of viſion nor the Sun's vertical, but between both; at the ſame time, the plane of the quadrant will not be vertical, and the altitude found by bringing the Sun's image to touch the horizon will not be the true altitude.

II. If the quadrant be held perpendicular to the horizon, and turned about upon a vertical axis, or one nearly ſo, the Sun will deſcribe an arch convex downwards in the fore-obſervation, and upwards in the back-obſervation, the motion of the Sun being the ſame way as the axis of viſion is carried in both caſes, and being to the angular motion of the quadrant, as the verſe-ſine of the Sun's altitude to the radius in the fore-obſervation, but as the verſe-ſine of the ſupplement of the Sun's altitude to 180° to the radius in the back-obſervation. The Sun therefore will move ſlower than the axis of viſion is the fore-obſervation, and conſequently will be left behind,

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with respect to the axis of vision, or seem to move backwards; and the Sun will move quicker than the axis of vision in the back-observation, or will seem to get before it. When the motion of the Sun in this arch is parallel to the horizon, the plane of the quadrant coincides with the vertical circle passing through the Sun, and consequently the quadrant is in a proper position for taking the Sun's altitude. But if the quadrant be held a little deviating from the perpendicular position to the horizon, and turned about upon an axis, either vertical or only nearly so, the arch described by the Sun apparently will cut the horizon, but will never move parallel to it, and consequently the quadrant will not be brought into a proper position for observing the Sun's altitude.

III. If the quadrant be turned on the line going to the Sun as an axis, the reflected Sun will be kept constantly in the axis of vision, and will describe an arch of a parallel circle about the real Sun, with a velocity which is to the angular motion of the quadrant, as the sine of the Sun's altitude is to the radius; and when the motion of the reflected Sun is parallel to the horizon, the quadrant is vertical.

Hence naturally arise the three methods of taking an altitude, which have been mentioned before. In the first, the axis of vision is supposed always directed to one and the same part of the horizon, namely that which is in the Sun's vertical. In the second, the observer is required to hold the quadrant truly vertical, and to turn himself upon a vertical axis;

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but it is evident neither of these motions can be accurately performed. In the third method, the observer is only required to move both himself and the quadrant, so as to keep the Sun always in or near the axis of vision, which may be performed very well, because the axis of vision is a visible and certain direction for it. One exception, however, should be made to this general rule, namely, in taking the Sun's altitude when very low, by the back observation; in which case it will be best to use the second method, or else to hold the quadrant perpendicular by judgment; which will be much facilitated by using a telescope containing wires in its focus parallel to the plane of the quadrant, as described in p. 106: for, in this case, the perpendicular position of the quadrant cannot be attained so near by the method of turning the quadrant on a line going to the Sun as an axis, as it can by the other method.

It remains to treat of the errors which may arise from a defect of parallelism in the two surfaces of the index-glass, and to point out the means of obviating them in the celestial observations. It is well known, that if a pencil of parallel rays falls upon a glass whose two surfaces are inclined to one another, and some of the rays are reflected at the fore-surface, and others passing into the glass and suffering a reflection at the back-surface and two refractions at the fore-surface emerge again from the glass, these latter rays will not be parallel to those reflected at the fore-surface, as they would have been if the surfaces of the glass had been parallel, but will be inclined to the same. I find that the angle of their mutual inclination, which may be called the deviation of the

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rays reflected from the back-surface, will be to double the inclination of the surfaces of the glass (which is here supposed to be but small), as the tangent of the angle of incidence out of air into glass, is to the tangent of the angle of refraction. Hence, in rays falling near the perpendicular, the deviation will be about three times the inclination of the surfaces; and if the angles of incidence be 50°, 60°, 70°, 80° or 85°, the deviations of the reflected rays will be about 4, 5, 7, 13 or 26 times the inclination of the surfaces, respectively. Had the deviation been the same at all incidences of the rays on the index-glass, no error would have been produced in the observation; because the course of the ray would have been equally affected in the adjustment of the instrument, as in the observation. But, from what has been just laid down, this is far from being the case, the deviation increasing according to the obliquity with which the rays fall upon the index-glass; so that in very oblique incidences of the rays, such as happen in measuring a large angle by the fore-observation or a small angle by the back-observation, the least defect in the parallelism of the planes of the two surfaces of the index-glass may produce a sensible error in the observation.

What is here said only takes place in the fullest extent, if the thickest or thinnest edge of the index-glass, or, to express the same thing in other words, the common section of the planes of the surfaces of the index-glass stands perpendicular to the plane of the quadrant; but, if the common section of the planes is inclined to the plane of the quadrant, the error arising from the defect of the parallelism of the

ſurfaces will be leſſened in the proportion of the ſine of the inclination to the radius; ſo that at laſt, when the common ſection becomes parallel to the plane of the quadrant, the error intirely vaniſhes. For this reaſon, Mr. Hadley very properly directed the thickeſt and thinneſt edges of the index-glaſs to be placed parallel to the plane of the quadrant. But, as it may well be queſtioned whether this care is always taken by the inſtrument-maker, and it cannot be ſuppoſed that the glaſſes can be ground perfect parallel planes, it would certainly be an advantage acquired to the inſtrument, could the error ariſing from a want of paralleliſm of the planes be removed in whatever poſition the common ſection of the planes ſhould be placed with reſpect to the plane of the quadrant. This will be effected for celeſtial obſervations, if the upper part of the index-glaſs be left unſilvered on the back, and made rough and blacked, the lower part of the glaſs being ſilvered as uſual, which muſt be covered whenever any celeſtial obſervations are made. Then, if the teleſcope be ſufficiently raiſed above the plane of the quadrant, it is evident that the obſervations will be made by the rays reflected from the fore-ſurface of the upper part of the index-glaſs, and conſequently, if the quadrant be adjuſted by making uſe of the ſame part of the index-glaſs, the obſervations will be true whether the two ſurfaces of the index-glaſs are parallel planes or not. The Sun or Moon may be thus obſerved by reflection from the unſilvered parts of the index-glaſs and horizon-glaſs, ſo that a paler darkening glaſs will ſuffice, and they will appear much diſtincter than from an index-glaſs wholly ſilvered with a deeper darken-

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ing glaſs; for although the ſurfaces of a glaſs may be parallel, yet there always ariſes ſome little confuſion from the double reflection. Neither will the Moon appear too weak by two unſilvered reflections, even when her creſcent is very ſmall, except ſhe ſhould be hazy or clouded; and then the light may be increaſed by lowering the teleſcope ſo as to take in part of the ſilvered reflection of the index-glaſs, which in this caſe muſt be uncovered: the ſame is alſo to be underſtood with reſpect to the Sun, ſhould his light be too much weakened by hazineſs or thin clouds. The horizon-glaſſes ſhould be adjuſted, or the error of adjuſtment found by the Sun or Moon; the firſt will be in general the beſt object for the purpoſe; and, as the Sun or Moon ſeen directly through the unſilvered part of the horizon-glaſs will be much brighter than the image of the ſame ſeen by two unſilvered reflections, it muſt be weakened by a darkening glaſs placed beyond the horizon-glaſs, the reflected image being farther weakened, if neceſſary, by a paler darkening glaſs placed in the uſual manner between the index-glaſs and the horizon-glaſs.

If a quadrant was deſigned principally for taking the diſtance of the Moon from the Sun and fixed ſtars, and was not wanted for obſerving terreſtrial angles, it would be the beſt way to have none of the glaſſes ſilvered, but to leave the horizon glaſſes intirely tranſparent, and to put a red glaſs for an index-glaſs of the ſame matter with the darkening glaſſes, which would reflect light from the fore-ſurface only.

The Sun's altitude might alſo be obſerved with this inſtrument, either by the fore or back-obſervation; and

and the altitude of the Moon might be taken with it in the night. But the altitudes of ſtars could not be obſerved with it, nor the Moon's altitude in the day time, which would however be no great inconvenience, as theſe obſervations might be well enough ſupplied by common quadrants.

The following rules for the ſize of the glaſſes and the ſilvering them, and the height of the teleſcope may be of uſe. The index glaſs and two horizon-glaſſes ſhould be all of equal height, and even with one another in height both at top and bottom. The teleſcope ſhould be moveable parallel to itſelf nearer to or farther from the plane of the quadrant, and the range of its motion ſhould be ſuch that its axis when at the loweſt ſtation ſhould point about $\frac{1}{10}$th of an inch lower than the top of the ſilvering of the horizon-glaſſes, and when at the higheſt ſtation ſhould point to the height of the middle of the unſilvered part of the index-glaſs. The height of the glaſſes, and the quantity of parts ſilvered and parts unſilvered, ſhould vary according to the aperture of the object-glaſs, as in the following table; where the firſt column of figures ſhews the dimenſions in parts of an inch anſwering to an aperture of the object-glaſs of $\frac{3}{10}$ths of an inch in diameter; the ſecond column what anſwer to an aperture of the object-glaſs of $\frac{4}{10}$ths of an inch in diameter; and the third, what are ſuitable to an aperture of the object-glaſs of $\frac{5}{10}$ths of an inch in diameter.

Diameter

	Parts of an Inch		
Diameter of aperture of object-glafs	,30	0,40	0,50
Height of glaffes	,90	1,13	1,37
Height of filvered part of index-glafs	,50	0,63	0,77
Height of unfilvered part of ditto	,40	0,50	0,60
Height of filvered part of horizon-glaffes	,25	0,35	0,42
Height of unfilvered part of ditto	,65	0,80	0,95

If the telefcope has a common object-glafs, the firft aperture of $\frac{3}{10}$ths of an inch will be moft convenient; but if it has an achromatic object-glafs, one of the other apertures of $\frac{4}{10}$ths or $\frac{5}{10}$ths of an inch, will be moft proper. The field of view of the telefcope fhould be 5 or 6 degrees, and the objects fhould be rendered as diftinct as poffible throughout the whole field, by applying two eye-glaffes to the telefcope. The breadth of the glaffes fhould be determined as ufual, according to the obliquity with which the rays fall on them and the aperture of the object-glafs.

I fhall conclude this paper with fome eafy rules for finding the apparent angular diftance between any two near land objects by the Hadley's quadrant.

To find the angular diftance between two near objects by the fore-obfervation. Adjuft the fore-horizon-glafs by the object intended to be taken as the direct object; and the angle meafured by the fore-obfervation on the arch of the quadrant between this object and any other object feen by reflection will be the true angle between them as feen from the centre of the index-glafs. But, if the quadrant be already

already well adjusted by a distant object, and you do not chuse to alter it by adjusting it by a near one, move the index, and bring the image of the near direct object to coincide with the same seen directly, and the number of minutes by which (o) of the index stands to the right hand of (o) of the quadrant upon the arch of the excess is the correction, which added to the angle measured by the arch of the quadrant between this direct object and any other object seen by reflection will give the true angular distance between them reduced to the centre of the index-glass.

To find the angular distance between two near objects by the back-observation.

It is supposed that the-horizon-glass is truly adjusted; if it is not, let it be so. Observe the distance of the objects by the back-observation, and take the supplement of the degrees and minutes standing upon the arch to 180 degrees, which call the instrumental angular distance of the objects; this is to be corrected as follows. Keep the centre of the quadrant or index-glass in the same place as it had in the foregoing observation, and observe the distance between the near object, which has been just taken as the direct object, and some distant object, twice; by making both objects to be the direct and reflected ones alternately, holding the divided arch upwards in one case and downwards in the other, still preserving the place of the centre of the quadrant. The difference of these two observations will be the correction, which added to the instrumental angular distance,

diſtance, found as above in the firſt obſervation between the firſt object and any other object ſeen by reflection, will give the true angular diſtance between them reduced to the centre of the index-glaſs.

But if you ſhould happen to be in a place where you cannot command a convenient diſtant object, the following method may be uſed.

The back-horizon-glaſs being adjuſted, find the inſtrumental angular diſtance between the objects; this is to be corrected by means of the following operations. Set up a mark at any convenient diſtance oppoſite or nearly ſo to the object which has been taken as the direct object; and looking at the direct object move the index of the quadrant, and bring the image of the mark to coincide with the direct object, and read off the degrees and minutes ſtanding on the arch of the quadrant, which ſubtract from 180 degrees, if (0) of the index falls upon the quadrantal arch; but add to 180 degrees, if it falls upon the arch of exceſs; and you will have the inſtrumental angular diſtance of the object and mark. Invert the plane of the quadrant, taking care at the ſame time not to change the place of its centre, and looking at the ſame direct object as before, move the index of the quadrant, and bring the image of the mark to coincide again with the direct object, and read off the degrees and minutes ſtanding on the arch, and thence alſo find the inſtrumental angular diſtance of the object and mark. Take the ſum of this and the former inſtrumental angular diſtance; half of its difference from 360° will be the correction, which added to the inſtrumental angular diſtance firſt found between the ſame direct object and the other object ſeen

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by reflection will give the true angular diſtance between them reduced to the centre of the index-glaſs.

It is to be obſerved, that if the mark be ſet up at the ſame diſtance from the quadrant as the direct object is, there will be no occaſion to invert the plane of the quadrant, but the obſerver need only make the image of the mark coincide with the direct object, then turn himſelf half round, and now taking the mark for the direct object cauſe the image of the former direct object to coincide with the mark, the divided arch of the quadrant being kept upwards, and the place of the centre of the quadrant remaining alſo the ſame in both caſes: half the difference of the ſum of the two inſtrumental angles from 360 degrees will be the correction of the adjuſtment as before.

Should only one of the objects be near, and the other remote (that is to ſay, half a mile diſtant or more) let the diſtant object be taken for the direct one, and the near object for the reflected one; and the true diſtance of the objects as ſeen from the centre of the index-glaſs will be obtained without requiring any correction, whether it be the back or fore obſervation that is made uſe of; only obſerving, as uſual, to take the ſupplement of what is ſhewn upon the arch to 180 degrees in the back-obſervation.

XV. *Account*